Utility Maximization with Neutral and Bad Goods

1. Consider a consumer whose utility function is

$$U(x_1, x_2) = x_1$$

where good x_2 is neutral. The prices are $p_1 = 5$ and $p_2 = 2$ and the income is $M = 50$. Determine the optimal consumption bundle and graph the budget constraint along with the indifference curve passing through the optimal bundle.

2. Consider a consumer whose utility function is

$$U(x_1, x_2) = x_1 - 2x_2$$

where good x_2 is a bad since its consumption reduces utility. The prices are $p_1 = 3$ and $p_2 = 4$ and the income is $M = 60$. Determine the optimal consumption bundle and graph the budget constraint along with the indifference curve passing through the optimal bundle.

Solution

1. Since the consumer derives utility only from x_1 the budget constraint is

$$5x_1 + 2x_2 = 50$$

To maximize utility the consumer spends all income on x_1 and sets $x_2 = 0$

Thus the optimal consumption bundle is

$$x_1^* = \frac{50}{5} = 10 \quad x_2^* = 0$$

The graph below illustrates the budget line and the indifference curve representing the consumer's preferences. Note that the indifference curve is the vertical line $x_1 = 10$ (since utility depends only on x_1) and it touches the budget line at the optimal bundle $(10, 0)$.

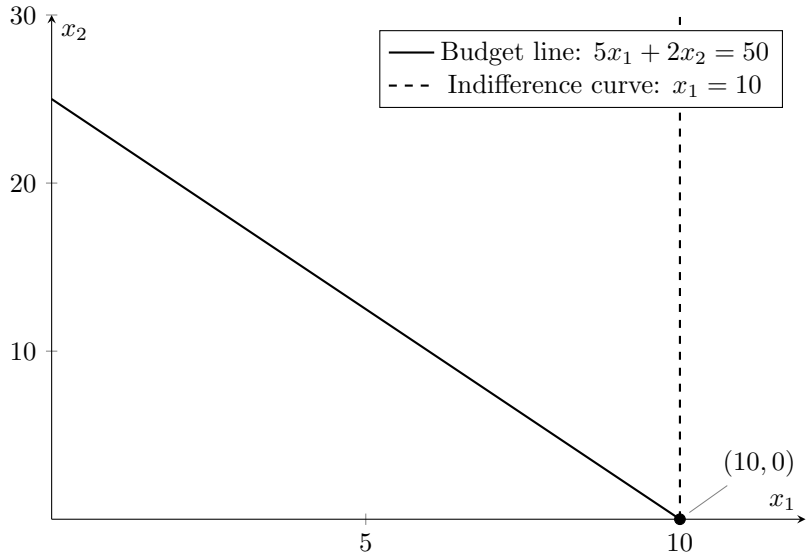

Figure 1: Graph for Neutral Good

2. Because consuming x_2 lowers utility the consumer will avoid it The budget constraint is

$$3x_1 + 4x_2 = 60$$

To maximize utility the consumer sets $x_2 = 0$ and allocates all income to x_1

Thus the optimal consumption bundle is

$$x_1^* = \frac{60}{3} = 20 \quad x_2^* = 0$$

The graph below shows the budget line and the indifference curve for which utility is constant. For the indifference curve we have

$$U(x_1, x_2) = x_1 - 2x_2 = 20,$$

which can be rewritten as

$$x_1 = 20 + 2x_2.$$

These two lines intersect at the optimal bundle $(20, 0)$.

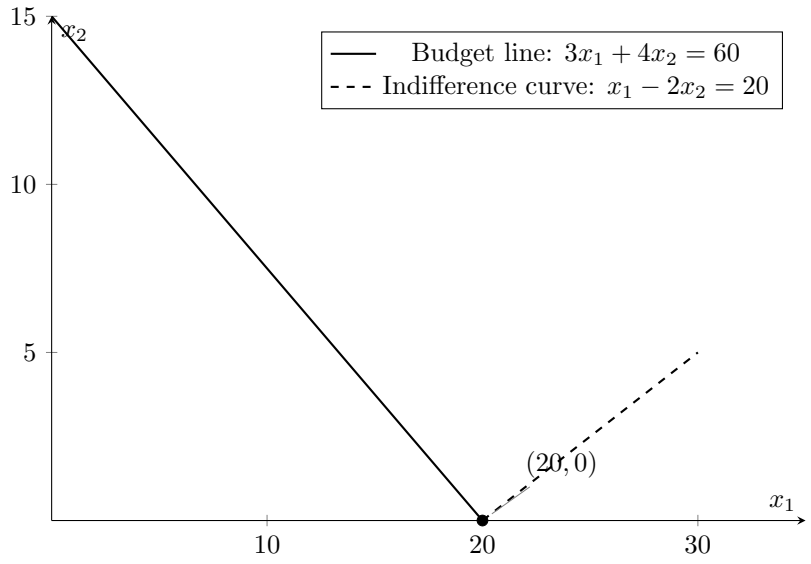

Figure 2: Graph for Bad Good